Capítulo 7. INTRODUCCIÓN AL BALANCE DE POBLACIÓN

7.1. Introducción. Analogía con reactores químicos

En la Figura 7.1 se presenta un esquema de un reactor químico donde ingresa y egresa una mezcla multicomponente. Si se realizarán experimentos sería posible medir, entre otras cosas, las composiciones de los reactivos y productos a la entrada y salida del reactor.

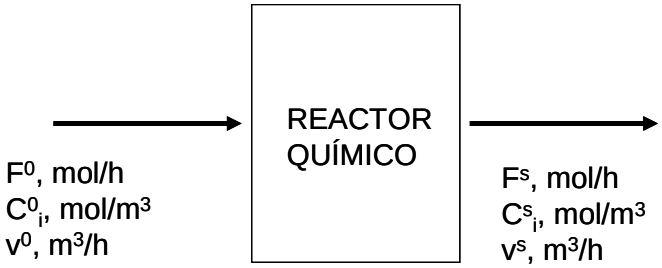


Figura 7.1. Reactor químico continuo con alimentación multicomponente.

Supongamos que se lleva a cabo la siguiente reacción:

$$A + B \rightarrow C + D \tag{7.1}$$

y que se conoce las composiciones de los reactivos y productos tal como se muestra en la Tabla 7.1. La misma información tabular la podemos representar gráficamente tal como se muestra en la Figura 7.2.

Tabla 7.1. Composiciones a la entrada y salida del reactor.

Entrada		Salida	
C^0_i	%molar	C^s_i	% molar
A	80	A	60
B	20	B	0
C	0	C	20
D	0	D	20

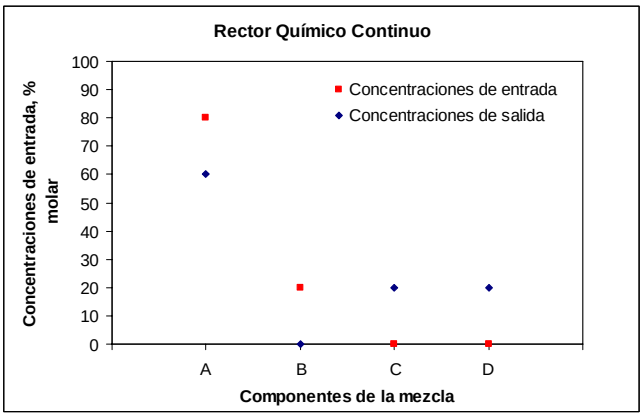


Figura 7.2. Representación gráfica de la Tabla 7.1

Para poder predecir las concentraciones de salida, conociendo las de entrada, es necesario establecer:

- Tipo de reacción química, estequiometría y cinética.
- Tipo de reactor/ tipo de flujo (TAC; RT; BATCH, etc.)
- Datos geométricos de la unidad de reacción, datos variables operativas de entrada y propiedades de la mezcla.

Con esta información es posible plantear los balances molares por componente, el balance de energía y cantidad de movimiento, y establecer así todas las variables a la salida del reactor.

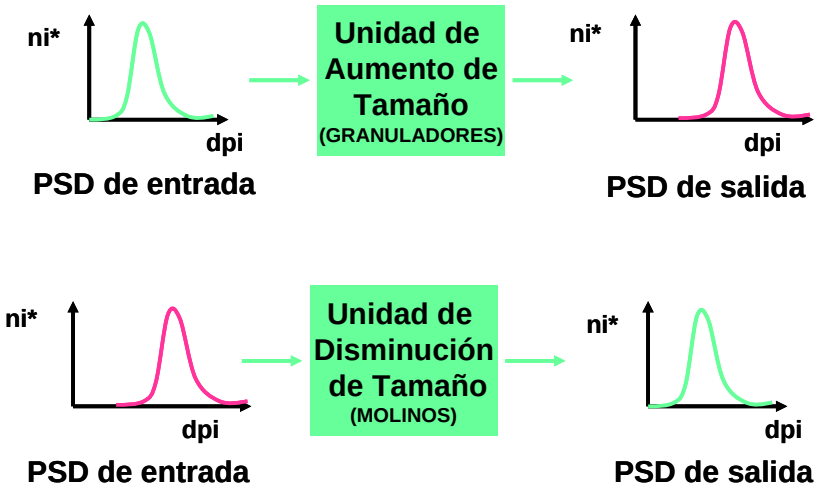


Figura 7.3. Granuladores y Molinos.

La Figura 7.3 muestra la operación de granuladores y molinos. Básicamente los granuladores son equipos en los cuales se aumenta el tamaño de partículas de la distribución entrante a la unidad (ver Tabla 7.2). Hay diferentes tipos de granuladores los cuales será presentados en el próximo capítulo. Un ejemplo de granulador sería el equipo descripto para recubrir los rocklets. Por su parte, como es conocido por todos, los molinos (en el capítulo 9 serán presentados algunos ejemplos) reducen el tamaño de las partículas que ingresan al equipo (ver Tabla 7.3). Un ejemplo que se puede citar es la molienda de trigo para la obtención de harina. La pregunta que surge es **¿qué ecuación constitutiva nos permite establecer la calidad del producto de estos equipos, en función de las variables operativas y de diseño?.** Así como los balances molares por componente, conociendo propiedades del sistema, permiten establecer la distribución de productos a la salida del reactor, el **BALANCE DE POBLACIÓN** es la herramienta que nos permite calcular distribuciones de tamaño de partículas (en inglés, Particle Size Distributions- PSDs) a la salida de un equipo dado.

Tabla 7.2. PSDs entrada/ salida granuladores.

Entrada		Salida	
d_{pi}	$n_i^* \times 100/N_T$	d_{pi}	$n_i^* \times 100/N_T$
0	0	0	0
1	20	1	20
2	60	2	60
3	20	3	20
4	0	4	0

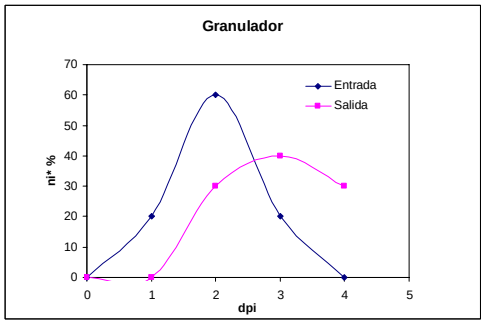
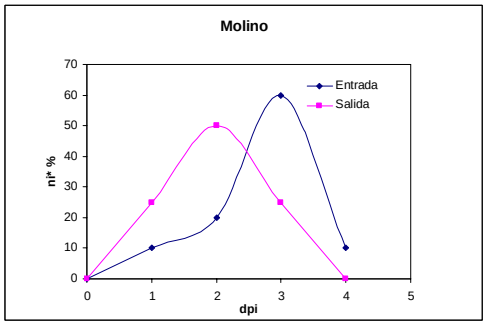


Tabla 7.3. PSDs entrada/ salida molinos.

Entrada		Salida	
d_{pi}	$n_i^* \times 100/N_T$	d_{pi}	$n_i^* \times 100/N_T$
0	0	0	0
1	10	1	25
2	20	2	50
3	60	3	25
4	10	4	0



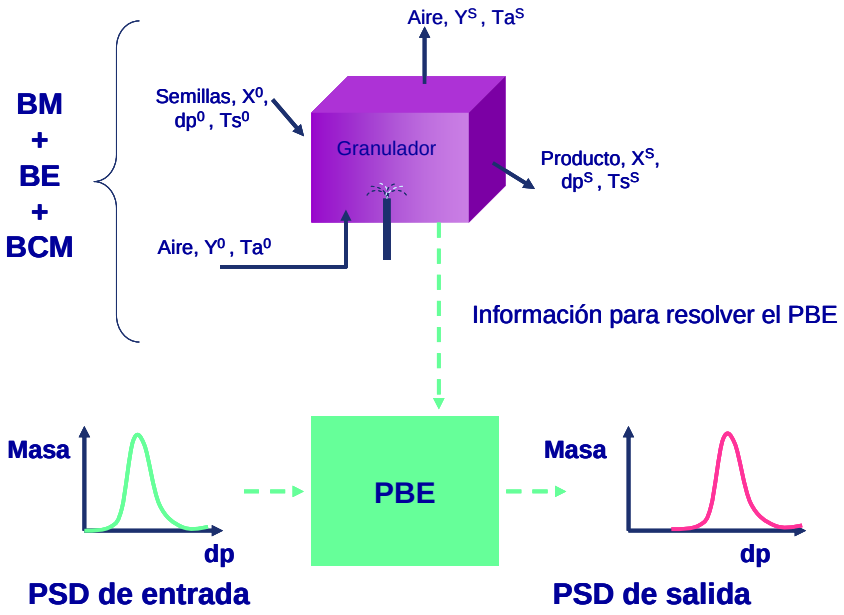


Figura 7.4. Balances necesarios para definir completamente un granulador.

En síntesis si quisiéramos modelar un granulador, por ejemplo uno de lecho fluidizado como el que se muestra en la Figura 7.4, se requeriría plantear los balances de masa, energía, cantidad de movimiento y por último el balance de población. Así como los balances molares requieren de información cinética, patrón de flujo, datos geométricos, etc. para describir las composiciones a la salida de una unidad de reacción, el balance de población (en inglés, Population Balance Equation – PBE) requiere de información cinética para describir la performance de unidades de aumento o reducción de tamaño.

7.2. Deducción del Balance de Población

Como texto para la deducción del balance de población utilizaremos la siguiente publicación que se adjunta al apunte: "*Teaching Population Balances for ChE Students: Application to granulation processes*", Chemical Engineering Education, Vol. 41, No. 3, 209-217, Summer 2007.

TEACHING POPULATION BALANCES FOR CHE STUDENTS:
Application to Granulation Processes

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In many chemical engineering degree programs worldwide, particle and powder technology is not afforded the same attention in the curricula as processes and technologies incorporating liquids and gases.[1] Consequently, it is not surprising that plants handling solids perform less optimally than those processing only liquids and gases.[2] There is therefore need for new courses in particle science and technology in the established chemical engineering curricula.

Chemical engineers are used to handle mass, energy, and momentum balances in modeling and designing equipment for the chemical industry. Often, however, they are not as familiar with the population balance equation (PBE) to describe important attributes of particulate streams (*e.g.*, particle size distributions). While the PBE is generally agreed to be difficult to solve, many students find even formulating it to be very complicated.

An optional course for education in particle technology has been introduced in the last year of the chemical engineering program at the Universidad Nacional del Sur, Argentina. During instruction of the PBE formulation, we found that analogies with chemical reaction principles (well known by the alumni) helped the students to understand this "new" constitutive equation.

In this work we are particularly focused on the approach to teaching PBE formulation in the context of granulation processes.

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THE NEED FOR PBE MODELING OF THE GRANULATION PROCESS

Figure 1 shows a schematic diagram of a granulation unit. The seeds—*i.e.,* particles whose diameters are smaller than those of the granular product—may be fed to the system either initially (batch processes) or continuously (continuous granulators). Inside the granulation "box," these particles undergo an effective growth. A liquid solution (*e.g.*, a concentrated solution or melt) is sprayed into the granulation unit. Depending on the type of granulator, air can be fed into the unit to fluidize the bed.[3]

Inside the granulator the particles are involved in many mechanisms of size enlargement or reduction. Depending on the process, they can occur alone or simultaneously. This is the reason why the seeds' particle size distribution (PSD) evolves to a different exit or final PSD (see Figure 1). The PBE is the tool that allows, for example, predicting the granules' size distribution.

GRANULATION RATE PROCESSES

The PBE formulation requires understanding of the phenomena the particles are subjected to during the granulation evolution. Figure 2 schematically shows the nucleation, layering, coalescence, attrition, and breakage processes. All these processes are well explained by, among others, Litster, et al.,[3] and Rhodes.[4]

Nucleation is the formation of new seeds from liquid or fine powder feed. New granules can be formed when the liquid drops (produced in the spray) solidify before they reach the surface of the seeds. This mechanism is discrete. This adjective means that the new nuclei just appear, *i.e.*, they are not produced gradually. *Layering* increases the granule size by coating the particle surface with drops produced in the spray zone. The growth is differential, *i.e.,* the particle size augments progressively. Particles may also undergo *coalescence*, *i.e.,* two particles agglomerate to give a bigger one, this being a discrete phenomenon. By means of the *attrition* mechanism the particles suffer surface wearing, a differential granule size reduction. The granule *breakage* is also a discrete process, where one particle can produce more than two fragments due to collisions with other particles and/or with the granulator walls.[4, 5]

The PBE has to capture all the granulation rate processes to predict the granulometry of the final product. Before dis-

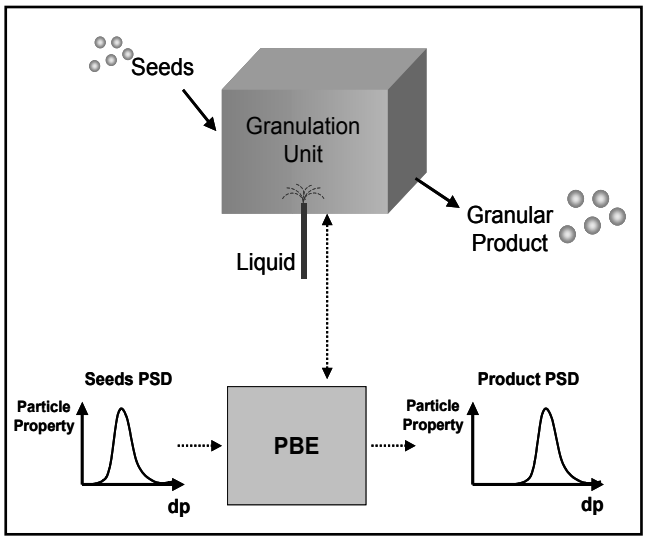


Figure 1. *A schematic diagram of a granulation unit and its relationship with the PBE.*

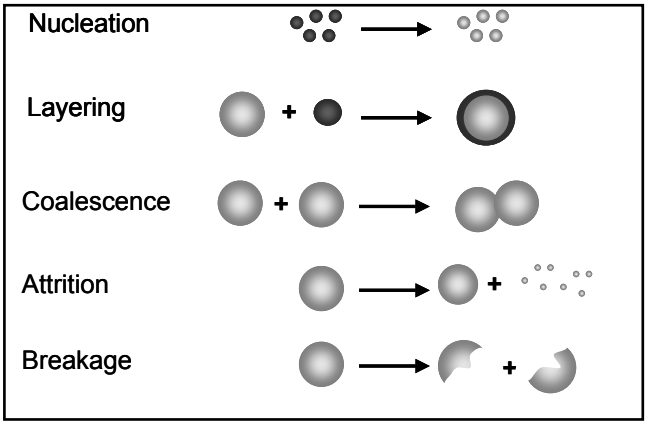


Figure 2. *Granulation rate processes.*

TABLE 1
Sieve Analysis of a Single Sample, Using Different Sets of Sieves

Fine Grid				Coarse Grid 1				Coarse Grid 2			
Size Range, mm	Count, #	n(dp), #/mm	dp_{av}, mm	Size Range, mm	Count, #	n(dp), #/mm	dp_{av}, mm	Size Range, mm	Count, #	n(dp), #/mm	dp_{av}, mm
0.00-0.21	0	0.00	0.11	0.00-0.297	0	0.00	0.15	0.00-0.21	0	0.00	0.11
0.21-0.297	0	0.00	0.25	0.297-0.59	30	102.39	0.44	0.21-0.42	10	47.62	0.32
0.297-0.42	10	81.30	0.36	0.59-1.19	90	150.00	0.89	0.42-0.84	50	119.05	0.63
0.42-0.59	20	117.65	0.51	1.19-2.00	160	197.53	1.60	0.84-1.68	160	190.48	1.26
0.59-0.84	30	120.00	0.72	2.00-3.36	40	29.41	2.68	1.68-2.38	90	128.57	2.03
0.84-1.19	60	171.43	1.02	3.36-4.76	0	0.00	4.06	2.38-4.76	10	4.20	3.57
1.19-1.68	100	204.08	1.44	4.76-5.66	0	0.00	5.21	4.76-5.66	0	0.00	5.21
1.68-2.00	60	187.50	1.84		N_T=320				N_T=320		
2.00-2.38	30	78.95	2.19								
2.38-3.36	10	10.20	2.87								
3.36-4.76	0	0.00	4.06								
4.76-5.66	0	0.00	5.21								
	N_T=320										

secting the population balance equation, it is necessary to understand how to represent particle populations properly. In the next section different particle size distributions (PSDs) will be discussed.

PARTICLE SIZE DISTRIBUTIONS

The particle size distributions are presented in many books, such as Litster, et al.,[3], Rhodes,[4] and Randolph and Larson.[6] The proper particle size distribution to compare populations is the density function as suggested, for example, by Litster, et al.[3] The students, however,—and even chemical engineers working in industry—frequently use the plot of number of particles (or mass fraction) as a function of mean diameters to represent the size distribution of a given particle population. In this section, the need for using the density function is presented through an example. This didactic explanation was found understandable by all the students that participate in the classes. The working example was sufficiently clear to prove the need for using the density function as a description of the particle size distribution.

Table 1 shows different data size analyses performed to the same particle sample. First, the population was analyzed employing 11 sieves (fine grid). Secondly, some sieves were extracted from the original set of sieves, and the same sample was studied by using just 6 sieves (coarse grid 1). Finally, the sample was analyzed by using 6 different sieves from those of coarse grid 1 (coarse grid 2). From this information, the histogram of frequency (count) vs. particle size can be built for the three grids presented in Table 1. Figure 3 shows the histograms that were thus obtained. Even though the same population has been analyzed, the histograms are very different. Therefore, the graphs of count vs. particle size cannot be used to evaluate size population similarities. The number of particles is often plotted as a function of the average diameter of the size interval. As can be seen in Figure 4, an inspection of the curves does not indicate that the distributions are similar, in agreement with the histograms of frequency (Figure 3).

The density function (n) is a particle size distribution that allows comparison of populations,[3] and it is defined in a discrete form as:

$$n_i(dp) = \frac{n_i^*}{\Delta dp_i} \qquad (1)$$

where n_i^* is the number of particles between two contiguous sizes (dp_i and dp_{i+1}) and Δdp_i is the interval width ($dp_{i+1} - d_{pi}$). The continuous density function can be expressed as follows:

$$n(dp) = \frac{dN}{d(dp)} \qquad (2)$$

where N is the number cumulative distribution and has the units of number of particles (#). The density function n (#/L; where L indicates a generic length unit) represents the number of particles per unit of particle size. The continuous density function verifies the following equation:

$$\int_0^\infty n(dp)d(dp) = N_T \qquad (3)$$

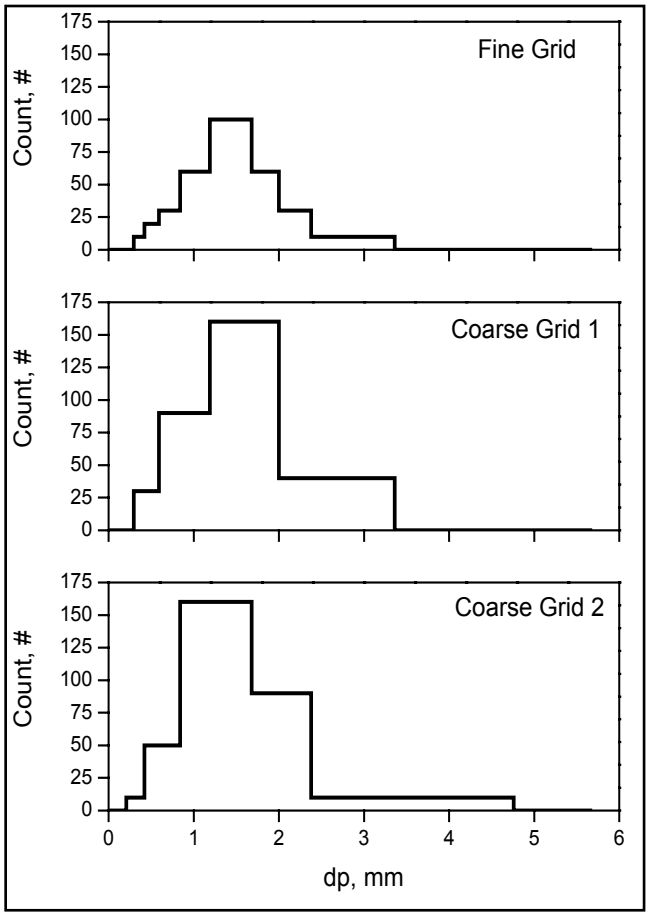


Figure 3. *Histogram of frequency (number) vs. particle size.*

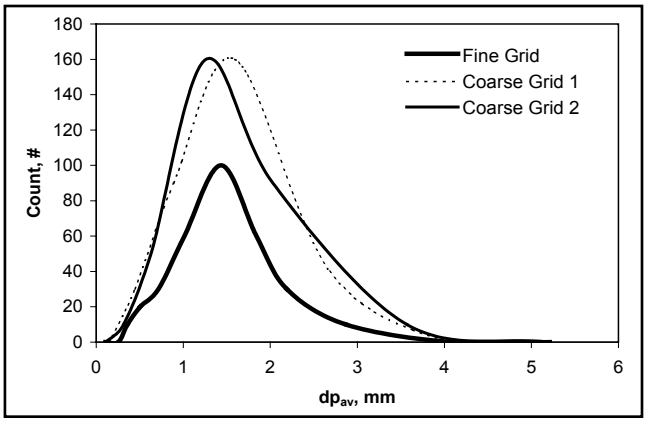


Figure 4. *Particle number as a function of the average diameter.*

Often the frequency or density distribution is expressed as a normalized distribution:

$$\int_0^\infty \frac{n(dp)}{N_T} d(dp) = \int_0^\infty f(dp) d(dp) = 1 \qquad (4)$$

where N_T and f(dp) are the total number of particles (#) and the normalized density function (L^{-1}), respectively.

Eq. (3) indicates that the total area under the curve n(dp) vs. dp has to be equal to the total number of particles. Figure 5 shows the histogram of the density function, calculated according to Eq. (1), for the fine grid presented in Table 1.

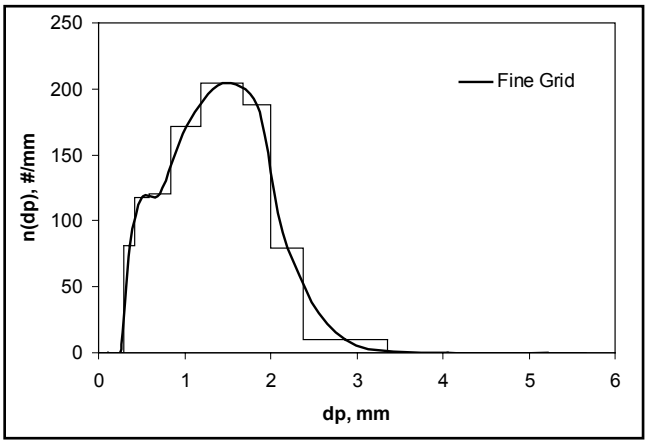


Figure 5. *Density function as a function of the particle diameter.*

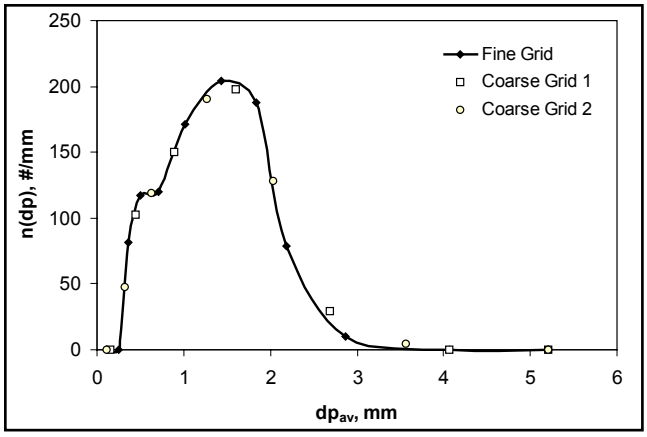


Figure 6. *Density function as a function of the average diameter for the sample analyzed with different sets of sieves.*

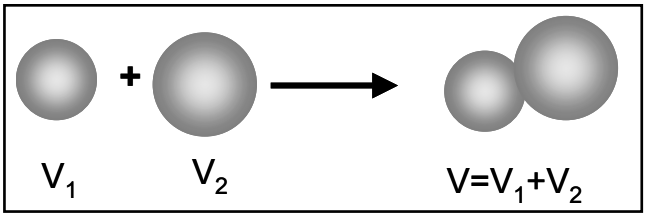


Figure 7. *Preservation of volumes in the coalescence process.*

Since Eq. (3) has to be satisfied, from the discrete representation (histogram) it is possible to obtain a continuous density function that preserves the area under the curve n(dp) vs. dp (this is the continuous curve that appears in Figure 5). As can be noticed, the original particle number attributed to a size interval in the histogram representation can be assigned approximately to the average diameter of the size interval for the continuous curve. That is the reason why the continuous density function is often plotted as a function of the arithmetic mean of the size range.

Figure 6 shows the number density function for the fine and coarse grids of Table 1. It is clear that the data of coarse grids 1 and 2 track well the density function calculated from the fine grid. This fact indicates that independent of the number of sieves employed in an experimental analysis or the grid points selected in a numerical procedure, the density function of a unique sample has to be equal. The density distribution is independent of the interval widths used for experimentation or numerical analysis. This point is very important, since the density function is commonly used to formulate the population balance equation.

When agglomeration takes place, particles of different volume coalesce to give a bigger one. As seen in Figure 7, the new particle has a volume equivalent to the sum of the volumes of the individual ones. For this reason, the PBE commonly uses the density function in terms of volume rather than particle diameter.[4, 7, 8] When the volume is selected as the representative size of the particles, the density function becomes:

$$n_i(Vp) = \frac{n_i^*}{\Delta Vp_i} \qquad (5)$$

where ΔVp_i is the volume interval width. For Eq. (5), the density function n has the units of $\#/L^3$.

At this point, the students should have in mind that the density function is the proper distribution for characterizing particulate systems and that the density function based on particle volume is appropriate to represent coalescence processes. Certainly, these items have been already treated in the literature[3, 5, 6]; nevertheless students often have difficulty understanding these two key points. The order of explanation of the topics was found adequate in the performed teaching experiences.

POPULATION BALANCE EQUATION

The ideal granulation units are classified according to their flow pattern into perfectly mixed and plug flow granulators. These are exactly analogous to the Continuous Stirred Tank Reactor (CSTR) and the Plug Flow Reactor (PFR) in chemical reactors, respectively.[3] The PBE for size enlargement processes has been introduced in several books and publications, among others, Litster, et al.,[3] Rhodes,[4] Randolph and

Larson,[6] and Heinrich, et al.[9, 10] Even though the PBE derivation can be found in the literature, the internal coordinates (particle properties) are hard to visualize for many students. In the following sections, the flow of particles in the internal coordinates is specifically discussed, focusing on the clear definition of the control volume to derive the PBE. Moreover, the granulation rate processes and flow of particles are compared with the chemical reactions and flow patterns occurring in chemical reactors, respectively. This didactic strategy, employed to some extent by Litster, et al.,[3] was found very useful in teaching the subject to advanced chemical engineering students.

PERFECTLY MIXED GRANULATORS

Figure 8 shows what can be seen through a window in a perfectly mixed granulator. Inside the unit, particles of different diameters are located everywhere. The perfectly mixed condition implies that the particle size distribution is identical throughout the granulator volume.

If the population of Figure 8, which is identical in any position of the granulator, is classified in baskets of different sizes, the particles contained in the granulator of Figure 8 can be schematized as shown in Figure 9.

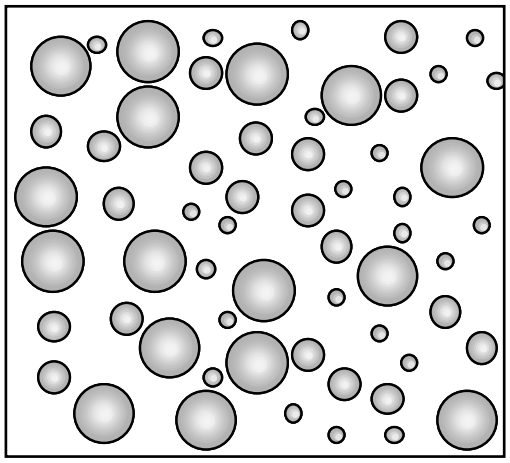
For a granulator like the one presented in Figure 1, a seed particle distribution is fed continuously to the system. In the "baskets" representation of the particle population inside the granulator (Figure 9), the addition contributes particles of different sizes that have to be classified in the size compartments. Similarly, a granular product extraction involves the removal of particles from different size boxes. In a common granulation process, the additions will specifically increase the number of particles in the smaller-size boxes.

If two particles coalesce to give a bigger one, in a discrete manner, both particles will abandon their respective size 'baskets" to increase the number of particles of a compartment that stores bigger particles. The effect of the binary breakage is opposite to the one caused by the agglomeration or coalescence phenomenon. Coalescence and breakage produce the birth of particles for some sizes and simultaneously the death of particles for other sizes. Nucleation leads to the birth of particles of small size. Even though it is not shown in Figure 9, the particle population generated by nucleation has a density function distribution. Therefore, the nuclei can enter several size compartments. Layering and attrition cause the differential particle enlargement and diminution. Therefore, the particles gradually leave the size clusters to the contiguous ones.[11]

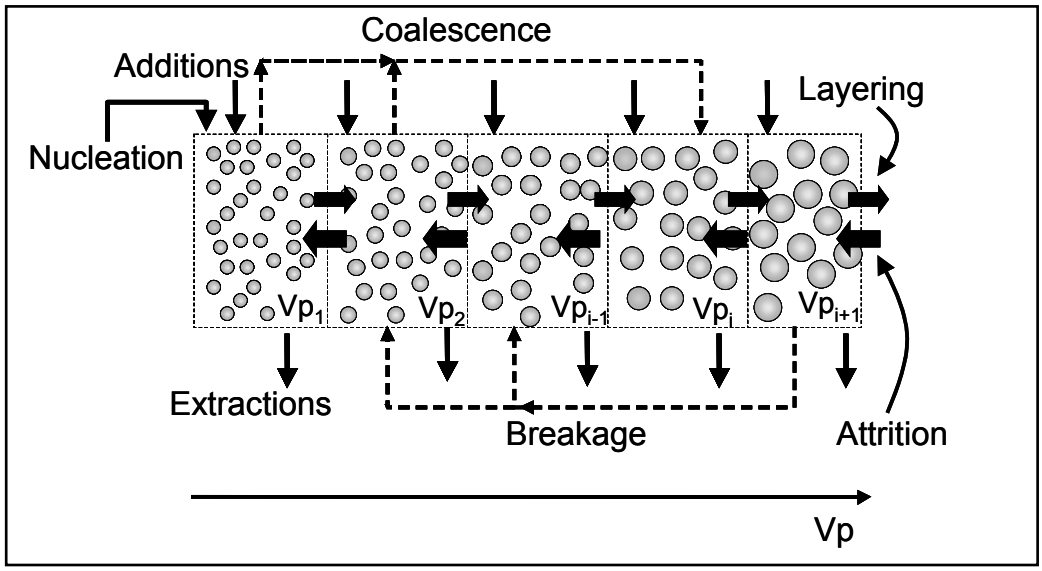
A similar conceptualization of the flow of particles shown in Figure 9 was introduced by Heinrich, et al.[9] In this paper, however, a complete description of all the feasible flows into and out of the particles "baskets" is given, facilitating the student's comprehension.

Once the influence of the granulation rate processes on the movement of particles from one size "basket" to others is understood, the PBE formulation can be discussed.

For the PBE derivation, the frequency distribution per unit of granulator volume (n^+) is also



▲ ***Figure 8.*** *Conceptualization of a perfectly mixed granulator.*



◀ ***Figure 9.*** *Classification of the particles according to the volume size.*

commonly used. This distribution is related to the density function n according to the following expression:

$$n^{+}(Vp) = \frac{n(Vp)}{V} \tag{6}$$

where V is the volume of the granulation unit. Therefore, $n^{+}(Vp)$ has $\#/L^{6}$ units.

The symbol $\dot{n}^{+}(Vp)$ is reserved for the frequency distribution per unit of volume and time ($\#/L^{6}$ t), and is used to represent birth and death rates; $\dot{n}^{+}(Vp)$ is completely equivalent to the chemical reaction rate of a chemical reactor. In order to understand these new properties better, Table 2 shows the analogy of the particle size distributions presented in this work with the variables commonly used in the design of chemical reactors.

Bearing in mind the concepts introduced in Figure 9, the particle number balance for a generic "basket" of volume ΔVp can be expressed as:

$$\begin{Bmatrix}\text{Number of particles}\\ \text{in time t}\end{Bmatrix} - \begin{Bmatrix}\text{Number of particles}\\ \text{in time t+}\Delta t\end{Bmatrix} + \begin{Bmatrix}\text{Number of particles "in"}\\ \text{by layering/attrition}\end{Bmatrix} - \begin{Bmatrix}\text{Number of particles "out"}\\ \text{by layering/attrition}\end{Bmatrix} + \begin{Bmatrix}\text{Number of particles "in"}\\ \text{by coales./break./nucleation}\end{Bmatrix} - \begin{Bmatrix}\text{Number of particles}\\ \text{"out" by coales./break.}\end{Bmatrix} + \begin{Bmatrix}\text{Number of particles}\\ \text{"in" by additions}\end{Bmatrix} - \begin{Bmatrix}\text{Number of particles}\\ \text{"out" by extractions}\end{Bmatrix} = 0 \tag{7}$$

$$\begin{aligned} &n^{+}(Vp)\ \Delta Vp\ V\big|_{t} - n^{+}(Vp)\ \Delta Vp\, V\big|_{t+\Delta t} \\ &+(G-A)n^{+}(Vp)V\ \Delta t\big|_{Vp} \\ &-(G-A)n^{+}(Vp)V\Delta t\big|_{Vp+\Delta Vp} \\ &+\dot{n}^{+}_{birth}(Vp)\ \Delta Vp\ V\Delta t - \dot{n}^{+}_{death}(Vp)\ \Delta Vp\, V\ \Delta t \\ &+Q_{in}\ n^{+}_{in}(Vp)\ \Delta Vp\ \Delta t \\ &-Q_{out}\ n^{+}_{out}(Vp)\ \Delta Vp\ \Delta t = 0 \end{aligned} \tag{8}$$

where G is the growth or layering rate (L^{3}/t); A, the attrition rate (L^{3}/t); $\dot{n}^{+}_{birth}(Vp)$, the birth of particles of class Vp by coalescence, breakage, and nucleation; $\dot{n}^{+}_{death}(Vp)$, the death of particles of class Vp by coalescence and breakage; Q_{in} and Q_{out} are the inlet and outlet particle volumetric flow rate (L^{3}/t); and $n^{+}_{in}(Vp)$ and $n^{+}_{out}(Vp)$ are the density distributions of the seeds and granular product streams ($\#/L^{6}$).

Dividing the entire resulting Eq. (6) by ΔVp and Δt and taking the limit as ΔVp and Δt tend to zero, the differential PBE for a perfectly mixed granulator is obtained:

$$\begin{aligned} &-\frac{\partial\left[n^{+}(Vp)V\right]}{\partial t} - \frac{\partial\left[(G-A)n^{+}(Vp)V\right]}{\partial Vp} \\ &+\dot{n}^{+}_{birth}(Vp)V - \dot{n}^{+}_{death}(Vp)V \\ &+Q_{in}n^{+}_{in}(Vp) - Q_{out}n^{+}_{out}(Vp) = 0 \end{aligned} \tag{9}$$

As mentioned above, $\dot{n}^{+}_{birth}$ and $\dot{n}^{+}_{death}$ are terms analogous to the chemical reaction terms in chemical reactors.[1] The birth and death coalescence and the breakage rates all require the use of theoretical or empirical models that depend on n^{+}, as the chemical reactions are functions of the concentration of different species.

Since the granulation is supposed to be perfectly mixed, the outlet density function of the population can be assumed to be equal to the distribution inside the granulator, $n^{+}_{out}(Vp) = n^{+}(Vp)$. Considering this relationship and Eq. (6), Eq. (9) becomes:

$$\begin{aligned} &-\frac{\partial n}{\partial t} - \frac{\partial\left[(G-A)n\right]}{\partial Vp} \\ &+\dot{n}_{birth} - \dot{n}_{death} \\ &+\frac{Q_{in}}{V}n_{in} - \frac{Q_{out}}{V}n = 0 \end{aligned} \tag{10}$$

A similar derivation of the PBE can be found elsewhere.[3,4] The sequential presentation of the process conceptualization, shown in Figure 9 and the PBE derivation, were found helpful while teaching, however. From our experience, this comprehensive view of the internal coordinates makes understanding Eq. (10) easier.

Eq. (10) has the following two first derivative terms: the accumulation term for unsteady state behavior, and the differential term of layering and attrition. The second first derivative represents a convective term in the Vp direction because there is a plug flow of particles in the volume particle axis. Therefore, a granulator that is perfectly mixed with respect

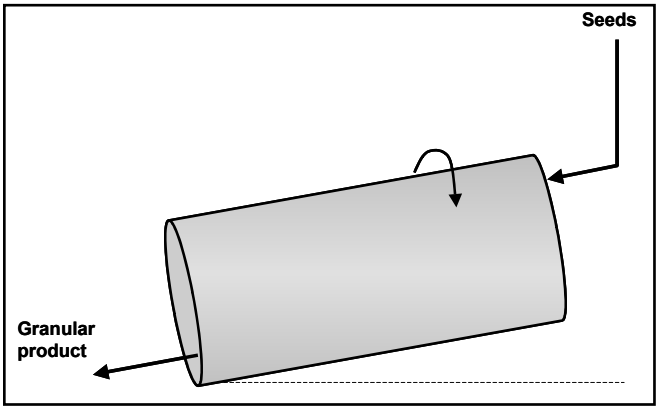


Figure 10. *Drum granulator scheme.*

to the real flow pattern exhibits a convective term in a new coordinate. The particle size is recognized as an internal coordinate; while the spatial coordinates of the equipment are commonly called external coordinates.[6, 12]

In short, a perfectly mixed granulator behaves as a plug flow for the chosen particle property the population balance is focused on. This analogy was found effective in teaching the process to advanced chemical engineering students, who are familiar with chemical reaction engineering. In the example described, the number variation was evaluated for changes in the particle volume. Other particles properties, however,—such as porosity and density—can also be considered. The size classification given in Figure 9 can be adapted to other particle properties for processes with several internal coordinates.

PLUG FLOW GRANULATORS

The drum granulators are basically inclined cylinders that are rotated to facilitate the movement of particles toward one end of the unit (see Figure 10). As a rough approximation, it can be assumed that the granules flow through the drum in plug flow.[3] In this section, the PBE is derived for this type of ideal granulator.

As is the case for chemical reactors, the flow of particles in plug flow granulators can be associated with a series of perfectly mixed units, as shown in Figure 11. For a Δz length element, the particles can also be classified according to their sizes, as schematized in Figure 12 (see page 216). The real flow of particles only occurs in the axial direction, though a flow of particles in the Vp direction is also needed in order to capture all the rate processes that can occur during granulation.

For a volume element $\Delta Vp\, A\, \Delta z$, where A is the granulator cross section (L^2), a number balance turns into:

$$\begin{Bmatrix}\text{Number of particles}\\ \text{in time t}\end{Bmatrix}-\begin{Bmatrix}\text{Number of particles}\\ \text{in time t+}\Delta t\end{Bmatrix}+\begin{Bmatrix}\text{Number of particles"in"}\\ \text{by layering/attrition}\end{Bmatrix}-\begin{Bmatrix}\text{Number of particles "out"}\\ \text{by layering/attrition}\end{Bmatrix}+\begin{Bmatrix}\text{Number of particles "in"}\\ \text{by coales./break./nucleation}\end{Bmatrix}-\begin{Bmatrix}\text{Number of particles}\\ \text{"out" by coales./break.}\end{Bmatrix}+\begin{Bmatrix}\text{Number of particles}\\ \text{"in" by convective flow}\end{Bmatrix}-\begin{Bmatrix}\text{Number of particles}\\ \text{"out" by convective flow}\end{Bmatrix}=0 \tag{11}$$

$$\begin{aligned} &n^+(Vp)\,\Delta Vp\;A\;\Delta z\big|_t - n^+(Vp)\Delta Vp\;A\Delta z\big|_{t+\Delta t}\\ &+(G-A)n^+(Vp)A\,\Delta z\Delta t\big|_{Vp} -\\ &(G-A)n^+(Vp)A\Delta z\Delta t\big|_{Vp+\Delta Vp}\\ &+\dot{n}^+_{birth}(Vp)\Delta Vp\,A\Delta z\Delta t\\ &-\dot{n}^+_{death}(Vp)\Delta VpA\;\Delta z\Delta t\\ &+v_z\,A\;n^+(Vp)\Delta Vp\;\Delta t\big|_z\\ &-v_z\;A\;n^+(Vp)\Delta Vp\;\Delta t\big|_{z+\Delta z}=0 \end{aligned} \tag{12}$$

where v_z is the velocity of the particles (L/t).

Dividing the entire resulting Eq. (12) by A, ΔVp, Δz and Δt and taking the limit as ΔVp, Δz and Δt tend to zero, the differential PBE for a plug flow granulator becomes:

$$-\frac{\partial n^+}{\partial t}-\frac{\partial\left[(G-A)n^+\right]}{\partial Vp}-\frac{\partial\left[v_z n^+\right]}{\partial z}+\dot{n}^+_{birth}-\dot{n}^+_{death}=0 \tag{13}$$

Eq. (13) shows the following three first derivative terms: the accumulation term, and two convective terms in the Vp (internal) and z (external) coordinates.

It is important to stress that the control volume to

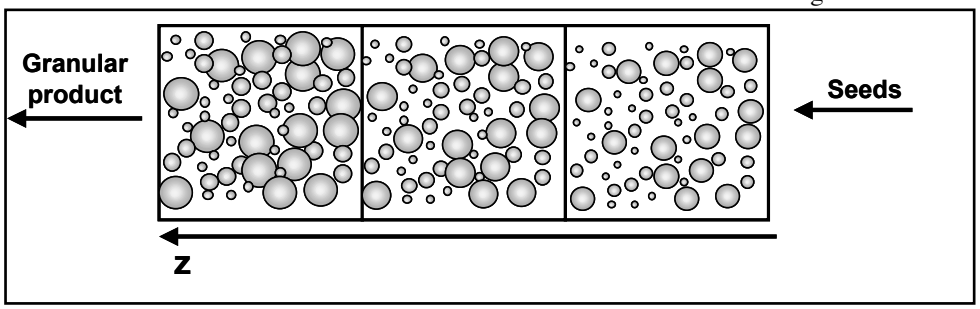


Figure 11. *Conceptualization of a plug flow of particles.*

TABLE 2
Analogy Between the Properties of Granulators and Chemical Reactors

Granulators			Reactors		
Symbol	**Description**	**Units**	**Symbol**	**Description**	**Units**
n	Frequency distribution	$\#/L^3$	N_A	Moles of the A specie	mol
$n+$	Frequency distribution per unit of volume	$\#/L^6$	C_A	Concentration	mol/L^3
$\dot{n}^+$	Frequency distribution per unit of volume and time	$\#/L^6\,t$	r_A	Reaction rate	$mol/L^3\,t$

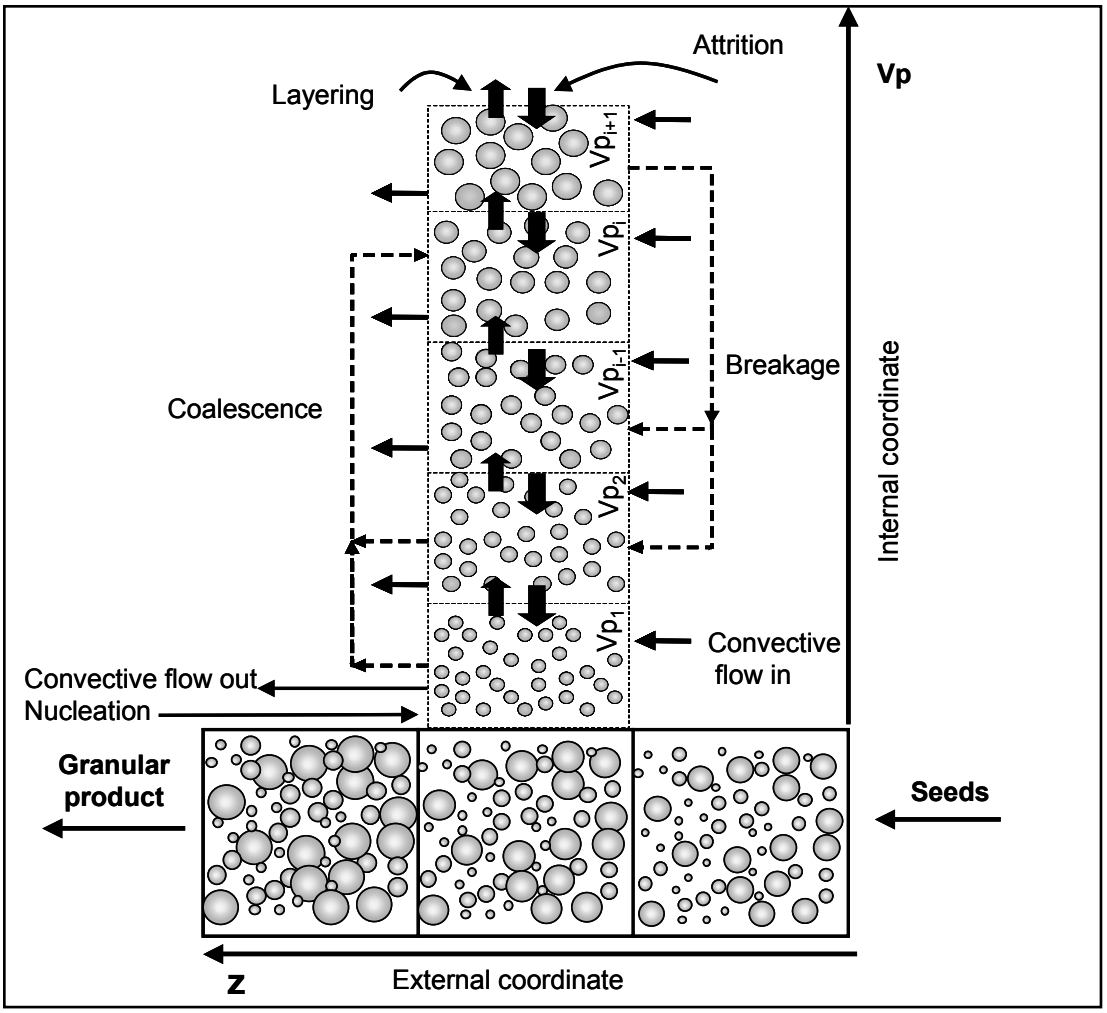


Figure 12. *Change of particle size in internal and external coordinates.*

derive the PBE for this system is not clearly presented in the literature. The conceptualization of the control volume given in Figure 12 lets the students understand in a rapid and easy way the PBE for plug flow granulators.

GENERALIZED PBE FOR IDEAL GRANULATORS

The PBE given by Eq. (13) can be generalized to a system where convective flow may occur in all the granulator real coordinates, considering that many particle properties may change during the granulation. Following the same line of reasoning as the one for plug flow granulator, the generalized PBE equation can be obtained[6]:

$$-\frac{\partial n^+}{\partial t}-\frac{\partial\left[v_x\, n^+\right]}{\partial x}-\frac{\partial\left[v_y n^+\right]}{\partial y}-\frac{\partial\left[v_z n^+\right]}{\partial z}$$
$$-\sum_{1}^{m}\frac{\partial\left[v_i\, n^+\right]}{\partial x_i}+\dot{n}^+_{birth}-\dot{n}^+_{death}=0 \qquad (14)$$

where v_x, v_y, and v_z are the velocities of the particles in the external coordinates x, y and z of the granulation unit; v_i corresponds to the rates of the selected particle properties that change in a differential manner; x_i are the internal coordinates, while m symbolizes the number of internal coordinates chosen to represent particle properties. The units of v_x, v_y, and v_z are L/t, while v_i has the units of the internal coordinates x_i per unit of time.

ABOUT THE TEACHING EXPERIENCES

As mentioned, the described material was used to teach the formulation of the population balance equation applied to granulation processes in the framework of an optional course given for advanced students of chemical engineering. The course is an elective in the last year of the chemical engineering career at the Universidad Nacional del Sur, Bahía Blanca, Argentina. The course entitled "Solids Processing" covers the following main topics: particle size analysis, particles in fluids, fluidization, solids conveying, gas/solid and solid separation, solids storage, solids caking, solids mixing, particle

size comminution and enlargement. The population balance equation is introduced when the last two chapters are covered. The approach described in this contribution is particularly useful for the last chapter (size enlargement). The emphasis on granulation processes is motivated by the presence of an industry that produces about 1 million tons of granulated urea/year in our city. It is important to note that the material presented in this work was also successfully employed to teach the "meaning and potential uses" of the PBE for granulation processes to engineers working at fertilizer plants.

The textbooks used in the course are, among others, the ones written by Rodhes,[4] Litster, et al.,[3] Seville, et al.,[13] and Kunii and Levenspiel.[14] The homework and exams assigned to the students include the development of the population balance equation for various granulation equipments and different particle properties. Regarding the solution of PBEs, only the simple cases for which analytical solutions can be found are covered in the undergraduate course.

CONCLUSIONS

The method presented in this paper for introducing the derivation of population balance equations was used to teach the subject both to chemical engineers that work in a granulation plant and to chemical engineering students. The conceptualization of the internal coordinates was clear and comprehensible for all of them. Our approach can be considered valuable as a teaching strategy to explain the PBE in the context of granulation processes. In particular, the analogy with chemical reactors is useful because this is a subject well known by the advanced chemical engineering students. Probably the PBE is not as extensively used as it could be, due to early difficulties understanding its derivation and then solving the final equation. The present contribution may help students/teachers that want to start learning/teaching population balances for model size enlargement processes.

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